Supplementary Information for "Asymmetry in the seasonal cycle of Antarctic sea ice driven by insolation"

Roach, Eisenman, Wagner, Blanchard-Wrigglesworth and Bitz

Text S1: Mathematical analysis of NoIce_NoDiff model configuration

Equation (7) in the main text, which represents the NoIce_NoDiff configuration of the model, can be rewritten as

$$\frac{dT}{dt} = \frac{T_0 - T}{\tau} + \frac{a\,S'(t)}{c_w}, \tag{S1}$$

where $T_0 \equiv \frac{a\,s_0 - A + B\,T_f + F_b}{B}$ is the time average of the spun up model solution and $\tau \equiv \frac{c_w}{B}$ is the model response timescale. Here we have decomposed the forcing as $S = s_0 + S'(t)$, where s_0 is the annual mean and $S'(t) \equiv S - s_0$ is the time-varying departure.

Next we subtract the annual mean from both sides of Eq. (S1), which leads to

$$\frac{dT'}{dt} = -\frac{T'}{\tau} + \frac{a\,S'(t)}{c_w}, \tag{S2}$$

with $T' \equiv T - T_0$. Two asymptotic regimes of Eq. (S2) can be considered. If the timescale of variability of $S'(t)$ (and hence similarly of T') is a lot longer than τ, then $\frac{dT'}{dt} \ll \frac{T'}{\tau}$, and the dominant balance in Eq. (S2) is

$$0 \approx -\frac{T'}{\tau} + \frac{a\,S'(t)}{c_w}, \tag{S3}$$

so that the temperature is always in approximate steady-state with the forcing: $T' \approx \frac{a\,S'(t)\,\tau}{c_w} = \frac{a\,S'(t)}{B}$. On the other hand, if the timescale of variability of $S'(t)$ (and hence similarly of T') is a lot shorter than τ, then $\frac{dT'}{dt} \gg \frac{T}{\tau}$, and the dominant balance in Eq. (S2) is

$$\frac{dT'}{dt} \approx \frac{a\,S'(t)}{c_w}, \tag{S4}$$

so the temperature is always in approximate quadrature with the forcing.

Inserting the idealized model parameters, $\tau \equiv \frac{c_w}{B} = 4.7$ yrs. Meanwhile, $S'(t)$ is annually periodic, so its timescales of variation are shorter than 1 yr. Hence Eq. (S2) can be roughly approximated by the second asymptotic regime (S4), with the rate of change of T scaling with the anomaly of S from its annual-mean value. This implies that having a narrow summer insolation peak with $S > s_0$ and a wider winter insolation trough with $S < s_0$, as occurs in high latitudes, leads to a brief summer warming period and a longer winter cooling period.

Table S1 Default parameter values in the idealized climate model.

D	diffusivity for heat transport	0.625 $\mathrm{Wm^{-2}K^{-1}}$
A	outgoing longwave radiation when $T = T_f$	194.8 $\mathrm{Wm^{-2}}$
B	outgoing longwave radiation temperature dependence	2.1 $\mathrm{Wm^{-2}K^{-1}}$
c_w	ocean mixed layer heat capacity	9.8 $\mathrm{W\,yr\,m^{-2}K^{-1}}$
a_0	ice-free coalbedo at equator	0.7
a_2	ice-free coalbedo spatial dependence	0.1
a_i	coalbedo where there is sea ice	0.4
F_b	heat flux from ocean below	5 $\mathrm{Wm^{-2}}$
k	sea ice thermal conductivity	2 $\mathrm{Wm^{-1}K^{-1}}$
L_f	sea ice latent heat of fusion	9.5 $\mathrm{W\,yr\,m^{-3}}$
T_f	freezing point	0° C
s_0	IdealSol insolation at equator	420 $\mathrm{Wm^{-2}}$
s_1	IdealSol insolation seasonal dependence	338 $\mathrm{Wm^{-2}}$
s_2	IdealSol insolation spatial dependence	240 $\mathrm{Wm^{-2}}$

Table S2 Description of each idealized model simulation and values of the parameter A.

Configuration	Description	A for SHSol	A for IdealSol
Full	Default configuration	194.8 W $\mathrm{m^{-2}}$	195.0 W $\mathrm{m^{-2}}$
NoIce	Omitting ice albedo feedback and ice thermodynamic effects	201.6 W $\mathrm{m^{-2}}$	201.5 W $\mathrm{m^{-2}}$
NoIce_NoDiff	As in NoIce but also omitting horizontal heat transport	138.6 W $\mathrm{m^{-2}}$	143.6 W $\mathrm{m^{-2}}$

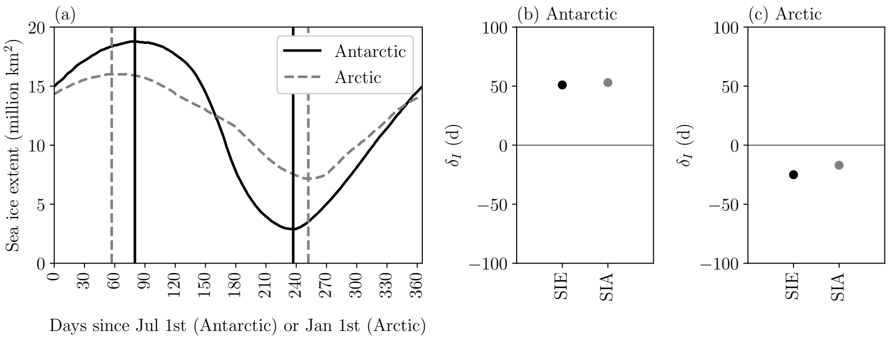

Fig. S1 Observed[21] 1979-1998 mean seasonal cycle of ice extent and ice area in both hemispheres. (a) As in Fig. 1 in the main text but for both the Arctic and the Antarctic. The time axis is shifted by half a year between the two curves to better align the times of maximum and minimum. (b) Seasonal asymmetry (δ_I) in Antarctic sea ice extent (SIE) and sea ice area (SIA). (c) As in panel (b) but for the Arctic.

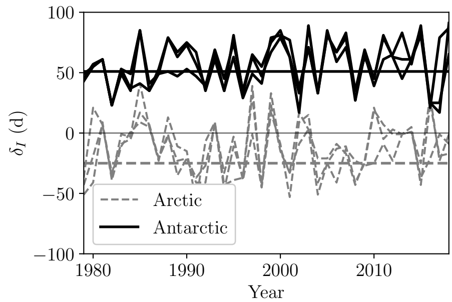

Fig. S2 Year-to-year variations in the value of δ_I in the Antarctic (solid) and Arctic (dashed) computed for each individual year during 1979-2018. For each hemisphere, three lines are plotted indicating the Climate Data Record (CDR), NASA Team, and Bootstrap sea ice datasets (all included with the CDR data release[21]). Horizontal lines show δ_I for the 1979-1998 mean seasonal cycle in the CDR dataset. The standard deviation of plotted year-to-year variability in the Antarctic CDR data is 18 days. When we compute the ice extent mean seasonal cycle averaged over each consecutive 20-year period during the entire 1979-2020 record, we find asymmetry values δ_I that vary between 49 and 79 days in the Antarctic and between -31 and 1 days in the Arctic.

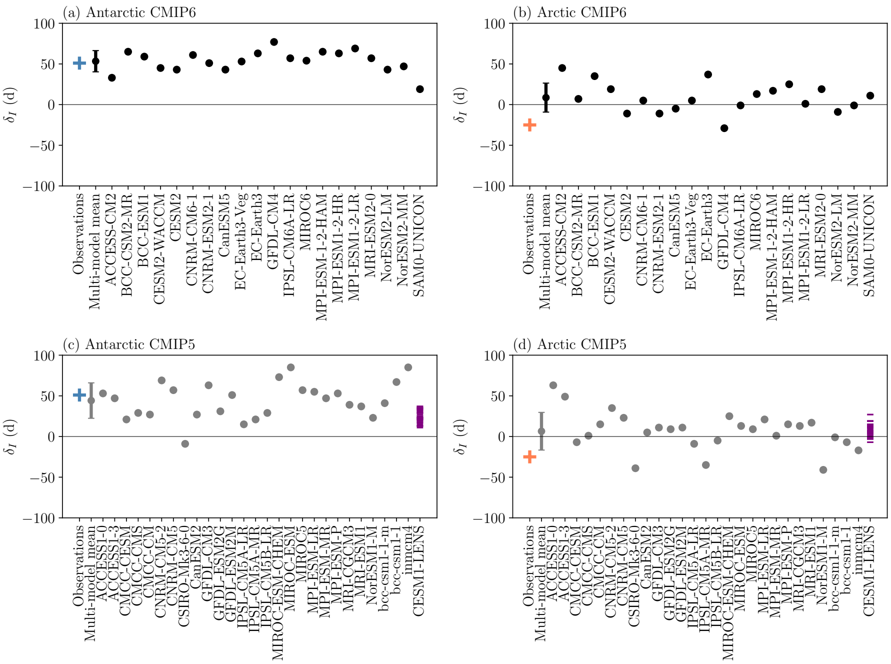

Fig. S3 Seasonal asymmetry (δ_I) in CMIP6 and CMIP5 simulations, as well as CESM1-LENS. (a) Equivalent to Fig. 2 in the main text. (b) As in panel (a) but for the Arctic. (c-d) As in panels (a-b) but for CMIP5 and CESM1-LENS.

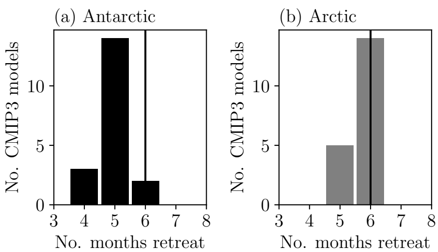

Fig. S4 Asymmetry in CMIP3 model results. (a) The number of months between the maximum and the minimum sea ice area in the simulated 1979-1998 mean seasonal cycle in 19 CMIP3 models. (b) As in panel (a) but for the Arctic.

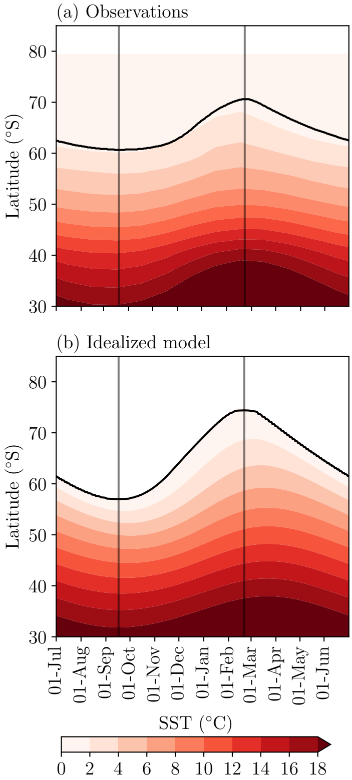

Fig. S5 Idealized model compared with observations. (a) Observed Southern Hemisphere sea surface temperature (colours) and zonal-mean ice edge latitude (black line). (b) As in panel (a) but using the simulation results from the Full configuration of the idealized model with SHSol forcing. Vertical lines indicate the times of maximum and minimum ice edge latitude, which correspond with the times of ice extent extrema. Here the observations are from the 1982-1998 mean annual cycle in NOAA OISSTv2 sea surface temperature[20] and NSIDC Climate Data Record sea ice extent[21], and the zonal-mean ice edge latitude is computed following ref 22 as the latitude with ocean area poleward of it equal to the ice extent, which is a quantity that maps monotonically from the ice extent. Note that evolution of the ice edge latitude in the idealized model broadly agrees with the observations, although the summer ice retreat in the model goes further poleward than the Antarctic continent permits in observations.

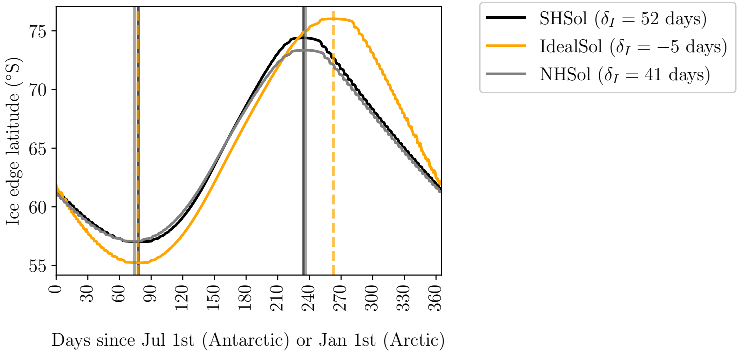

Fig. S6 Seasonal cycle in the ice edge latitude simulated with the Full configuration of the idealized model using IdealSol and SHSol insolation forcing, as well as Northern Hemisphere insolation forcing (indicated here as NHSol, using the same value of A as the SHSol run).

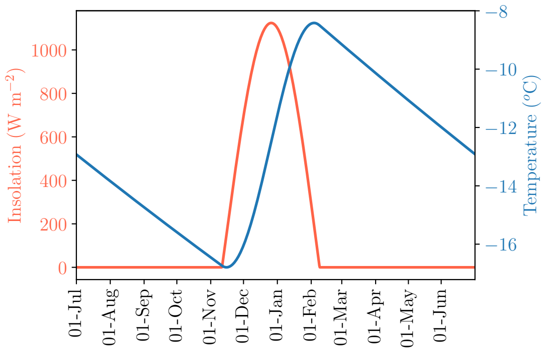

Fig. S7 Cartoon insolation field S and resulting temperature T simulated with Eq. (7) in the main text. Here the insolation field is constructed from the insolation at 90^{o}S by making the period that is not polar night half as long and twice as bright in order to exaggerate the brevity of the mid-summer insolation peak, and the values of A and a are as in the SHSol case at 64.5°S. Note the extreme seasonal asymmetry in the temperature response, with a brief period of warming and a long period of cooling.

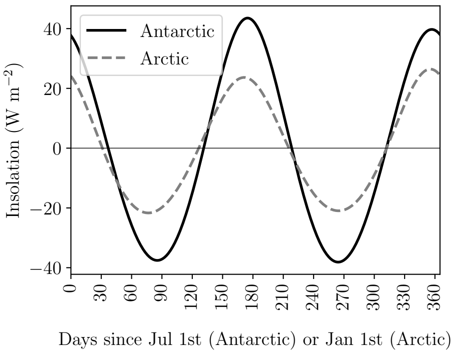

Fig. S8 As in Fig. 4b in the main text, but including the Arctic. This shows the difference between the top-of-atmosphere solar insolation and its annual harmonic ($S - S_1$) at 64.5° latitude in each hemisphere.

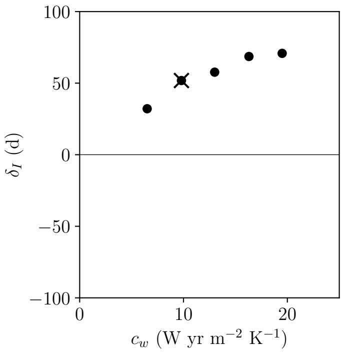

Fig. S9 Antarctic sea ice extent seasonal asymmetry in the Full version of the WE15 model with the ocean mixed layer heat capacity (c_w) varied between two thirds and double the default value (marked $\times$), which corresponds to a mixed layer depth of 75 m.

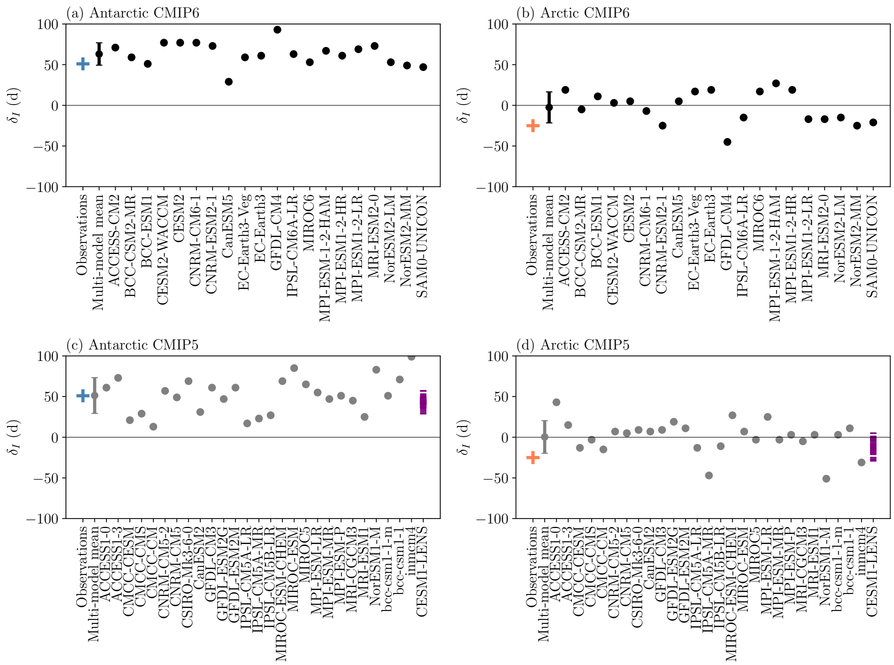

Fig. S10 As in Fig. S3, but using simulated ice extent rather than simulated ice area. Note that this figure and Fig. S3 both use the observed ice extent (see Methods).

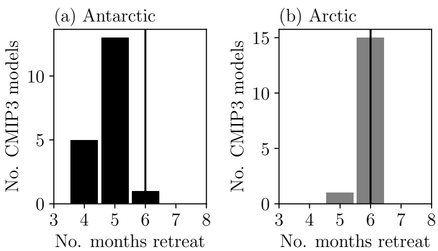

Fig. S11 As in Fig. S4, but using simulated ice extent rather than simulated ice area.